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LOOK FORS - ACTIONS FOR EFFECTIVE MATHEMATICS TEACHING AND LEARNING

Promoting Professional Reflection and Growth Through "Snapshot" Classroom Visits

Ongoing feedback is important in the professional growth of all educators. Administrators play a key role in this process for teachers. It is important that administrators develop and refine their mentorship skills in providing informative and reflective feedback to their teachers. Such informal dialogue promotes a consistent school-wide focus on developing a school culture of inquiry and learning of mathematics for teaching, through:

- promoting the use of effective mathematics instructional practices
- improving student achievement in mathematics
- making decisions about program interventions for students at risk
- building capacity for mathematics knowledge for teaching and instructional leadership in mathematics education
- directing appropriate resources to support student learning of mathematics

A "snapshot" is a short visit to a classroom to support teachers' learning. As administrators frequently visit classrooms, it is important to have a clear vision of effective mathematics teaching and learning.

The following five key messages describe a vision of effective mathematics teaching and learning:

- Key Message 1 – Effective mathematics learning environments are challenging, developmentally appropriate for all students, and strategically organized.

- Key Message 2 – Effective mathematics programs include problem-solving-based mathematical tasks and lessons that engage students at their level of readiness and provoke them to develop conceptual understanding, to generate, select, and use appropriate procedures / algorithms, and problem-solving strategies, and to activate multiple representations of their mathematical thinking.

- Key Message 3 – Effective mathematics instruction engages all students in developing deep conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, clear and precise mathematical communication, and a positive and productive disposition towards mathematics.

- Key Message 4 – Effective mathematics assessment is on-going and promotes continual growth in students' mathematics learning over time.

- Key Message 5 – Teachers need time and support to develop "pedagogical content knowledge"; that is, the knowledge of mathematics for teaching.

In this document, each key message is further described with student and teacher look-fors. These look-fors are sample actions that promote effective mathematics teaching and learning in classrooms. As well, the key messages are supported with questions (i.e., introductory, probing, next steps) that administrators can use to foster professional discussion.

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Five Key Messages for Mathematics Teaching and Learning

All students and teachers can learn significant mathematics within a nurturing learning environment, given sufficient time (i.e., period length and frequency), developmentally appropriate learning goals, well-considered learning materials, and strategic program, instruction, and assessment.

Key Message 1 – Effective mathematics learning environments are challenging, developmentally appropriate for all students, and strategically organized.	Within this learning environment, students learn mathematics by modelling and analyzing mathematical ideas using multiple representations (e.g., calculators, manipulatives, drawings, symbols, words), through interactive discussions, and by posing questions for clarification and further inquiry. In such social learning contexts, students develop and refine their thinking about mathematical ideas, strategies, and solutions. Mathematics learning in this environment fosters students' and teacher's curiosity, perseverance, and self-confidence.
Key Message 2 – Effective mathematics programs include problem-solving-based mathematical tasks and lessons that engage students at their level of readiness, provoke students to develop conceptual understanding, generate, select, and use appropriate procedures/ algorithms, and problem-solving strategies, and to activate multiple representations of their mathematical thinking.	Students learn mathematics through problem-solving-based lessons. Generally, lessons can be organized as one of three instructional designs: exploratory/investigation, guided, and modelled/direct instruction, chosen in relation to the type of mathematics to be learned: concepts, procedures/algorithms, strategies (e.g., mental math, problem solving). These lessons systematically include parts of the problem-solving process[3]: understand the problem, make a plan, carry out the plan, look back. As well, these lessons are organized to activate and build on students' prior, intuitive, and embodied knowledge of mathematics and to foster mathematical communication.
Key Message 3 – Effective mathematics instruction engages all students in developing deep conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, clear and precise mathematical communication, and a positive and productive disposition towards mathematics.	A mathematics program is only as effective as its implementation in instruction. Such effective mathematics instruction is framed around a problem-solving process. This teaching and learning process focuses on the students posing and/or understanding a mathematics problem, developing and analyzing several solutions. The teacher strategically coordinates students' sharing and reflections of their mathematical thinking, actions, and solutions to highlight the mathematics learning. Students consolidate their learning through shared and independent practice.
Key Message 4 – Effective mathematics assessment is on-going and promotes continual growth in student's mathematics learning over time.	Students learn mathematics within an assessment process which includes students demonstrating what they know and can do in different ways and students receiving constructive and focused feedback from their peers and teacher. Assessment is the process of gathering information from a variety of sources that accurately reflects how well a student is achieving the curriculum expectations. Assessment for learning is descriptive feedback to the student that guides efforts towards improvement and informs the teacher's daily program, instructional decisions, and actions. Assessments of learning are collections and records of the paths of students' achievement of expectations.
Key Message 5 – Teachers need time and support to develop "pedagogical content knowledge"; that is, the knowledge of mathematics for teaching.	This special form of knowledge bundles mathematical knowledge with knowledge of learners, mathematics learning process, and mathematics pedagogy. This mathematics knowledge for teaching mathematics (Ball, 2000, p. 245) is a "special amalgam of knowledge that links content and pedagogy. Included here is knowledge of what is typically difficult for students, or representations that are most useful for teaching a specific idea or procedures, and of ways to develop a particular idea."

Key Message 1 – Learning Environment - Effective mathematics learning environments are challenging, developmentally appropriate for all students, and strategically organized.		
Questions	**Classroom Look-Fors - Students are:**	**Classroom Look-Fors – Teachers are:**
Introductory • What aspects of your classroom learning environment support student learning and achievement in mathematics? **Probing** • How have you organized classroom time for students to learn mathematics every day? • How does your classroom (inside and outside) show the mathematics that students are learning? • How have you organized your classroom for shared and independent mathematics learning? • What ways can displayed student work be used for ongoing learning? • What ways have parents been involved in their children's learning of mathematics? • What ways have students and teachers from other classes (i.e., same grade, different grade) been involved in the children's learning of mathematics? • What evidence do you have that your students are developing greater confidence and interest in mathematics? **Next Steps** • What ways can your mathematics learning environment be enhanced to improve student learning and achievement in mathematics?	• sitting in flexible groupings - face to face in pairs or small groups, with sufficient space for group writing and use of manipulatives for learning • selecting and using learning materials (e.g., calculators, computer software, Internet Java applets, manipulatives, newspapers, textbooks) available in the classroom as thinking tools and for re-presenting mathematics • demonstrating mathematical thinking and doing in different ways (e.g., building, calculating, discussing, dramatizing, drawing, graphing, manipulating materials, questioning, sorting) • communicating ideas, solutions, and strategies using oral and written mathematical language (e.g., pictures, symbols, words) • "doing," collaboratively and independently, mathematics (e.g., develop plan and solution for a problem, explaining and analyzing student's solutions to a problem, describing a mathematical idea) • listening actively to other students and the teacher by asking questions, sharing ideas, strategies, and adapting their communication as the discussion ensues • referring to classmate's mathematical ideas and solutions that are posted on the classroom wall to explain and question mathematical ideas, algorithms, and strategies • taking risks to share ideas, solutions, and strategies, that the student knows are imprecise, in order to obtain feedback and suggestions from classmates and the teacher • sharing ideas, questions, and strategies that parents and students from other classes have about solutions to a problem, curiosities about the validity of a strategy or procedure/ algorithm demonstrated in class	• designating a visible mathematics areas in the room for student's mathematical exploration, practice, and/or problem solving (e.g., math table, math wall, math bulletin board) • organizing classroom furniture, so there is sufficient table space for small group and paired discussion, writing, and use of manipulatives for learning • organizing manipulatives (e.g., balance scales, base 10 blocks, interlocking cubes, calculators, geoboards, pattern blocks, square tiles, two colour counters) for easy student access and re-presenting mathematical concepts and processes • using manipulatives, children's literature, and media texts (e.g., newspapers, Internet, television clips) to set the context for solving lesson problems, to make plans to solve problems, and to carry out solutions to problems • modeling and promoting the use of visuals organizers, (e.g., arrays and grids, concrete graphs, five and ten frame, number charts, number lines) to represent ideas • communicating using precise oral and written mathematical language (e.g., pictures, symbols, words) • modeling and encouraging positive attitude, confidence, and perseverance when solving problems and making sense of student ideas and responses • circulating among students to listen to discussions for mathematical language use and watch their actions • prompting students to summarize, synthesize, and generalize their observations and calculations • recording strategically student ideas on a black/white board and referring to student work to consolidate, connect, or introduce mathematics (e.g., How are your solutions similar? Different? • displaying student work that reflects the mathematics that students are currently learning •

Key Message 2 – Curriculum Program - Effective mathematics programs include problem-solving-based mathematical tasks and lessons that engage students at their level of readiness, provoke students to develop conceptual understanding, generate, select, and use appropriate and procedures/algorithms, and problem-solving strategies, and to activate multiple representations of their mathematical thinking.		
Questions	**Classroom Look-Fors – Students are:**	**Classroom Look-Fors –Teachers are:**
Introductory • In what ways does your curriculum program support student learning and achievement of mathematics? **Probing** • How have you organized your program to enable student learning of all grade-level curriculum expectations? • How are your lessons designed for student learning of mathematical concepts, procedures/algorithms, and mental math strategies through problem solving? • What ways are the mathematics process skills explicit in your lesson plans? • In what ways are different mathematics learning materials used in your program? • What preparations have you made to anticipate students' prior mathematics knowledge, students' differentiated responses and knowledge, and their evolving mathematical thinking throughout the lessons? **Next Steps** • What aspects of your mathematics curriculum program could be improved to better support student learning and achievement in mathematics?	• demonstrating the mathematics they know and can do (i.e., from previous experiences, inside and outside of class), at the start of the lesson (e.g., through paired and small group discussion, solving a problem similar to previous day, playing a game) • understanding the mathematics details of a problem and making a plan to solve a problem • selecting concrete, visual, and electronic learning tools (e.g., calculators, computer software, manipulatives) to make their thinking explicit and/or to explore patterns • developing, selecting, and applying computational and problem-solving strategies as they develop solutions to problems and pose further "What If …" problems • developing and applying reasoning skills (e.g., pattern recognition, classification, recognition of relationships, use of counter-examples) to make and investigate conjectures and construct and defend arguments • creating a variety of representations of mathematical ideas, making connections among them, and applying them to solve problems (e.g., by using concrete materials, physical actions such as hopping or clapping, physical models, pictures, numbers, invented symbols, diagrams, graphs, onscreen dynamic representations) • learning in different groupings (e.g., whole class, small group, pairs; homogeneous, heterogeneous) throughout a lesson • reflecting on and monitoring their thinking to help clarify their understanding (e.g., by comparing and adjusting strategies used, by explaining why they think their results are reasonable, by recording their thinking in a math journal) • making connections among mathematical concepts and procedures, and relating mathematical ideas to situations or phenomena in other contexts (e.g., real-life, imaginary, music) • communicating mathematical thinking orally, visually, and in writing, using everyday language, grade-appropriate mathematical vocabulary, and a variety of representations and conventions	• including all curriculum expectations from the five mathematics strands in a school year-long program • identifying mathematics "knowledge packages" or interconnected concepts and skills for units of study • identifying the learning goals of lessons explicitly in day plans, curriculum unit plans, long range plans that relate to clusters of related curriculum expectations (i.e., Big Ideas, knowledge package) • planning clusters of lessons that helps students attain conceptual understanding and procedural fluency within problem-solving contexts • choosing teaching/learning strategies that activate students' prior knowledge so students are prepared cognitively, socially, and emotionally for new learning (e.g., through discussion, choosing a problem similar to previous day, presenting a math game focused on using previous day learning goals) • choosing a lesson problem that enables a range of entry points for students who demonstrate different levels of mathematics achievement • addressing different learning styles (e.g., auditory, kinesthetic, visual) in teaching/learning strategies outlined in the program • including flexible learning groups and providing time for independent thinking and shared discussion • choosing learning contexts that effectively and sensibly provoke reasons for learning the mathematics (e.g., mathematical contexts, physical contexts, real-life contexts) • selecting thinking tools that are appropriate for the mathematics content and processes that students are learning • organizing three-part lessons based on problem-solving (i.e., beginning, middle, end) so there is sufficient time for activating prior knowledge, new learning, reflecting, consolidation, and practice • incorporating different lesson design strategies (e.g., exploration/investigation, guided, modelled/direct instruction) as it relates to the mathematics learning goals (e.g., communication, concepts, procedures/ algorithms/strategies)

Key Message 3 – Classroom Instruction - Effective mathematics instruction engages all students in developing deep conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, clear and precise mathematical communication, and a positive and productive disposition towards mathematics.		
Questions	**Classroom Look-Fors - Students are:**	**Classroom Look-Fors – Teachers are:**
Introductory • What aspects of your classroom instruction support student learning and achievement of mathematics? **Probing** • How do you ensure that your students are learning mathematics through problem solving? • What does effective student learning of mathematics look like, sound like, and feel like throughout a lesson? • What kinds of questions do you and the students pose throughout the lesson? • What ways do you prepare to respond to a variety of students' solutions? • What strategies do you use to pace the students to ensure that they engage sufficiently in learning during the beginning, middle and end of a mathematics lesson? • What cooperative learning strategies are effective for prompting student individual thinking and collaborative? • How do you know that your instruction is making a difference for all students? **Next Steps** • What ways can your mathematics instruction be enhanced to improve student learning and achievement?	• showing curiosity, participating actively in the class learning community • monitoring and reflecting on their own thought processes by asking questions such as "what if I change that dimension?" • representing their thinking in many ways to develop a depth of understanding and see ideas from various points of view (e.g., graphic organizers, physical models, pictures, diagrams, pictographs) • reasoning and proving their ideas and solutions with examples, counter examples, and models • solving mathematical problems: gathering and analysing data, listening to explanations, reading text, justifying and defending a position in pairs or in small groups • persevering to solve mathematical problems (i.e., understand a problem, make a plan, carry out a plan, look back), to ask questions to clarify thinking, to develop and verify the utility of different strategies (e.g., procedures/ algorithms, mental math, solutions to a problem) • connecting different strands in mathematics (e.g., using their concept and skills from one strand of mathematics to understand concepts and skills in another strand) • thinking aloud to make explicit her/his own internal dialogue during problem solving • discussing with others the process by which they developed their solutions • communicating and adapting their thoughts with one another and the teacher both during and after the lesson (e.g., In the consolidation part of the lesson, the students turn to a partner and share a solution to the problem. e.g., "Did you get the same solution? Did you solve it in the same way?") • communicating so they can share, reflect upon, and clarify their ideas, solutions, and strategies • making sense of errors to clarify and deepen mathematical understanding •	• pacing the lesson so there is sufficient time for active student learning, reflection, consolidation, and practice • identifying learning goal of lessons through think-alouds, recording goal on the board, posting goals on a bulletin board • posing a lesson problem or investigation without giving students the steps to deriving a solution so that all students can engage at their own levels • prompting students to make conjectures, and justify solutions, orally and in writing • making the mathematics in student responses explicit (e.g. "You used a chart to clearly organize your combinations," instead of "good work") • prompting students to articulate their thinking, to listen to different perspectives of others, to adapt their thinking, and to refine their understandings • models think-aloud to make explicit her/his own internal dialogue during problem solving • making sense of a range of student thinking, in terms of precision in their use of mathematics, strategies, representations, and solutions to problems • choosing student solutions that show different representations of mathematical ideas and strategies • organizing student work samples for the class discussion to develop collective understanding of the lesson goals, to deepen individual knowing, and to support students' movement towards more precise and efficient methods • prompting students to continue to think about additional solutions to problems, to pose and solve similar problems for another day

Key Message 4 – Assessment - Effective mathematics assessment is on-going and promotes continual growth in student's mathematics learning over time.		
Questions	**Classroom Look-Fors – Students are:**	**Classroom Look-Fors –Teachers are:**
Introductory • What aspects of your classroom assessments support student learning and achievement of mathematics? **Probing** • How do you know what your students understand and can do in mathematics? • What strategies do you use to check students' consolidation of learning during a lesson and throughout a unit of study? • How do you monitor student learning throughout a chapter or unit of work to ensure equity of outcome? (e.g., equity does not mean equal time) • How do you monitor and respond to the mathematics learning of the class collective? • What ways do you adjust subsequent lessons based on the assessment you gather? • How are you planning, gathering, recording, and analyzing assessment data differently for diagnostic (initial), formative (assessment for learning) and summative (assessment of learning) assessments? **Next Steps** • What ways can your assessment practices be enhanced to support the improvement of student learning and achievement in mathematics?	• completing mathematical tasks and solving problems inside and outside of school, using purposeful and differentiated learning materials (e.g., pattern blocks, newspapers, computer) • identifying and discussing their strengths and areas needing improvement in understanding mathematics and using mathematical processes • demonstrating their mathematical understandings and queries through oral explanations, diagrams, concrete models, dramatizations, written words and symbols • making judgements about how well they understand the mathematics • making reasonable predictions about how well they did on a piece of written work or an assignment • participating positively in classroom interaction and showing pride in their mathematical ideas, solutions, and questions • asking for feedback from peers and teacher • listening actively to feedback from others, adapting their thought when the feedback deepens their learning and improves their achievement • rethinking their mathematical ideas, solutions, and strategies based on feedback from peers and teachers, in terms of using alternative computation and problem-solving strategies, checking for errors and omissions in the mathematics solutions that were identified and shared during class discussion	• designing ongoing review to monitor student's mathematics learning throughout a unit of study • identifying the learning needs of all students, in terms of content readiness, learning styles, including those who require accommodations and modifications to the regular program • identifying and accommodating program materials, content, and pace in relation to student readiness • using a variety of assessment strategies to gather information about student learning (e.g., assignments, day-to-day observations and conversations, demonstrations, projects, performances and tests) • observing and recording anecdotal comments about student's mathematical thinking and actions, as it relates to the learning goal of the lesson (e.g., curriculum expectations and mathematical processes) • interviewing or conferencing with small groups, pairs, and/or individual students to gain understanding of the student's achievement of the lesson goals (e.g., curriculum expectations and mathematical processes) throughout the lesson • collecting and maintaining individual student achievement records based on data gathered from observation, interview/ conference, performance tasks, analysis of student work samples • pointing out and highlighting students' different strategies while addressing the key concept/big idea or focus of the lesson • using strategies and tools to assess both the processes and products of mathematics learning • providing students with timely and descriptive feedback that guides improvement in their mathematical thinking and doing, during the lesson, and following up on homework • choosing next lesson goal and problem to build on the mathematics that students know and can do and to direct them toward making new connections to extend their knowledge • communicating clearly to parents and students the mathematics assessment criteria and the tools used to assess student learning and achievement in mathematics

Key Message 5 - Teacher Knowledge - Teachers need time and support to develop "pedagogical content knowledge"; that is, the knowledge of mathematics for teaching.		
Questions	**Classroom Look-Fors - Students are:**	**Classroom Look-Fors – Teachers are:**
Introductory • What mathematics knowledge, skill, and sensibilities do you rely on for planning and implementing your mathematics program? **Probing** • How did you use resource materials (e.g., teacher's guide for textbook, teacher resource book, MOE guides) to make sense of the mathematics for teaching? • Why were those mathematical models or representations necessary for student learning in the lesson? • Which mathematical responses were you expecting most often from the students? Least often? Why? • What mathematical issues emerged during the lesson? Why? • How did those mathematical issues inform your instructional decisions during the lesson? For the next lesson? • What do you need to understand about the mathematics you are teaching before the next lesson? How will you find out? • How do you learn more mathematics for your daily teaching? **Next Steps** • What ways can your mathematics pedagogical knowledge be further developed to support improvements in student learning and achievement?	• constructing mathematical understanding using their own knowledge and feedback from peers and the teacher	• understanding the mathematics "knowledge package" or interconnected mathematics concepts and skills for a unit of study • anticipating the range of student responses and the mathematical schema they might have about the mathematics in the lesson and prepares to address them • being aware of the possible strategies students might use to solve a lesson problem and of the mathematics that underlie these strategies • listening and watching for students' demonstration of specific understandings, organizing responses in a productive way, and prompting students to analyse and evaluate the different points of view • posing questions that are thought provoking and capture the essence of the mathematics for student learning • using precise and carefully chosen language to discuss the mathematics and requiring the same from students • responding to students' questions and offering clarification and next steps to their thinking and solutions • providing consolidation and practice that is appropriate to student learning from the lesson • providing mathematics tasks, questions, and feedback that provoke students to clarify and deepen their mathematical understanding • understanding how students' mathematical ideas, queries, and solutions to problems are related to one another and to the mathematics "knowledge package"